

SOLUTIONS FOR A SECOND-ORDER DELAY DIFFERENTIAL INCLUSION ON THE HALF-LINE WITH BOUNDARY VALUES

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Abstract. In [15], Wei solved a delay differential equation on the half-line. The current paper is an extension of these results to the set-valued case. The results involve measurable selections and the contraction mapping theorem for set-valued functions.

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