Differential
Equations
& Applications
Volume 9, Number 4 (2017), 553–563
doi:10.7153/dea-2017-09-38

MULTIPLE POSITIVE SOLUTIONS FOR A NONLINEAR CHOQUARD EQUATION WITH NONHOMOGENEOUS

HONG-YAO LI, CHUN-LEI TANG AND XING-PING WU

Abstract. In this paper, we study the existence of multiple positive solutions for the following equation:

$$-\Delta u + u = (K_\alpha(x) * |u|^p)|u|^{p-2}u + \lambda f(x), \quad x \in \mathbb{R}^N,$$

where $N \geqslant 3$, $\alpha \in (0,N)$, $p \in (1+\alpha/N, (N+\alpha)/(N-2))$, $K_\alpha(x)$ is the Riesz potential, and $f(x) \in H^{-1}(\mathbb{R}^N)$, $f(x) \geqslant 0$, $f(x) \not\equiv 0$. We prove that there exists a constant $\lambda^* > 0$ such that the equation above possesses at least two positive solutions for all $\lambda \in (0, \lambda^*)$. Furthermore, we can obtain the existence of the ground state solution.

Mathematics subject classification (2010): 26D15, 26A51, 32F99, 41A17.
Keywords and phrases: Positive solutions, nonlinear Choquard's equation, mountain pass lemma, Ekeland's variational principle.

REFERENCES

[1] E. H. LIEB AND B. SIMON, *The Hartree-Fock theory for Coulomb systems*, Comm. Math. Phys. **53**, 3 (1977), 185–194.
[2] E. H. LIEB, *Existence and uniqueness of the minimization solution of Choquard's nonlinear equation*, Stud. Appl. Math. **57**, 2 (1977), 93–105.
[3] P. L. LIONS, *The Choquard equation and related questions*, Nonlinear Anal. **4**, 6 (1980), 1063–1072.
[4] J. MAWHIN AND M. WILLEM, *Critical point theory and Hamiltonian systems*, Applied Mathematical Sciences **74**, Springer-Verlag, New York, 1989.
[5] R. PENROSE, *On gravity's role in quantum state reduction*, Gen. Relativ. Gravit. **28**, 5 (1996), 581–600.
[6] Z. J. ZHANG, *Multiple solutions of nonhomogeneous for related Choquard's equation*, Acta Math. Sci. Ser. B Engl. Ed. **20**, 3 (2000), 6–12.
[7] T. KÜPPER, Z. J. ZHAN AND H.Q. XIA, *Multiple positive solutions and bifurcation for an equation related to Choquard's equation*, Proc. Edinb. Math. Soc. **(2) 46**, 3 (2003), 597–607.
[8] N. ACKERMANN, *On a periodic Schrödinger equation with nonlocal superlinear part*, Math. Z. **248**, 2 (2004), 423–443.
[9] Z. J. ZHANG, T. KÜPPER AND A. L. HU, H.-Q. XIA, *Existence of a nontrivial solution for Choquard's equation*, Acta Math. Sci. Ser. B Engl. Ed. **26**, 3 (2006), 460–468.
[10] J. WEI AND M. WINTER, *Strongly interacting bumps for the Schrödinger-Newton equations*, J. Math. Phys. **50**, 1 (2009), 012905, 22 pp.
[11] S. CINGOLANI, S. SECCHI AND M. SQUASSINA, *Semi-classical limit for Schrödinger equations with magnetic field and Hartree-type nonlinearities*, Proc. Roy. Soc. Edinburgh Sect. A **140**, 5(2010), 973–1009.
[12] L. MA AND L. ZHAO, *Classification of positive solitary solutions of the nonlinear Choquard equation*, Arch. Ration. Mech. Anal. **195**, 2 (2010), 455–467.
[13] S. CINGOLANI, M. CLAPP AND S. SECCHI, *Multiple solutions to a magnetic nonlinear Choquard equation*, Z. Angew. Math. Phys. **63**, 2 (2012), 233–248.
[14] S. CINGOLANI, M. CLAPP AND S. SECCHI, *Intertwining semiclassical solutions to a Schrödinger-Newton system*, Discrete Contin. Dyn. Syst. Ser. S **6**, 4 (2013), 891–908.

Paper DEA-09-38

[15] M. CLAPP AND D. SALAZAR, Positive and sign chaning solutions to a nonlinear Choquard equation, J. Math. Anal. Appl. **407**, 1 (2013), 1–15.
[16] V. MOROZ AND J. VAN SCHAFTINGEN, *Ground states of nonlinear Choquard equations: existence, qualitative properties and decay asymptotics*, J. Funct. Anal. **265**, 2 (2013), 153–184.
[17] V. MOROZ AND J. VAN SCHAFTINGEN, *Nonexistence and optimal decay of supersolutions to Choquard equations in exterior domains*, J. Diff. Eqns. vol. **254**, 8 (2013), 3089–3145.
[18] C. O. ALVES AND M. B. YANG, *Existence of semiclassical ground state solutions for a generalized Choquard equation*, J. Diff. Eqns. vol. **257**, 11 (2014), 4133–4164.
[19] D. F. LÜ, *A note on Kirchhoff-type equations with Hartree-type nonlinearities*, Nonlinear Anal. **99**, 99 (2014), 35–48.
[20] S. CINGOLANI AND S. SECCHI, *Semiclassical analysis for pseudorelativistic Hartree equations*, J. Diff. Eqns. vol. **258**, 12 (2015), 4156–4179.
[21] D. F. LÜ, *Existence and concentration of solutions for a nonlinear Choquard equation*, Mediterr. J. Math., **12**, 3 (2015), 839–850.
[22] V. MOROZ AND J. VAN SCHAFTINGEN, *Groundstates of nonlinear Choquard equations: Hardy-Littlewood-Sobolev critical exponent*, Commun. Contemp. Math., **17**, 5 (2015), 1550005–1550017.
[23] V. MOROZ AND J. VAN SCHAFTINGEN, *Semi-classical states for the Choquard equation*, Calc. Var. Partial Differential Equations, **52**, 1–2 (2015), 199–235.