EXISTENCE AND MULTIPLICITY RESULTS FOR THE FRACTIONAL p–LAPLACIAN EQUATION WITH HARDY–SOBOLEV EXPONENTS

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Abstract. In this paper, we investigate the following fractional p-Laplacian problem

$$\begin{cases} (-\Delta)^s_p u = \lambda |u|^{p-2}u + \dfrac{|u|^{p^*_{s,\alpha}-2}u}{|x|^{\alpha}} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain containing the origin in $\mathbb{R}^N$ with Lipschitz boundary, $p \in (1,\infty)$, $s \in (0,1)$, $0 \leqslant \alpha < ps < N$ and $p^*_{s,\alpha} = (N-\alpha)p/(N-ps)$ is the fractional Hardy-Sobolev exponent. We prove the existence, multiplicity and bifurcation results for the above problem. Our results extend some results in the literature for the fractional p-Laplacian problem involving critical Sobolev exponent and the p-Laplacian problem involving Hardy-Sobolev exponents.

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