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EXISTENCE OF POSITIVE SOLUTIONS FOR NONLINEAR FRACTIONAL NEUMANN ELLIPTIC EQUATIONS

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Abstract. This article is devoted to study the fractional Neumann elliptic problem

$$\begin{cases} \varepsilon^{2s}(-\Delta)^s u + u = u^p & \text{in } \Omega, \\ \partial_\nu u = 0 & \text{on } \partial\Omega, \\ u > 0 & \text{in } \Omega, \end{cases}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, $N > 2s$, $0 < s \leqslant s_0 < 1$, $1 < p < (N+2s)/(N-2s)$, $\varepsilon > 0$ and ν is the outer normal to $\partial\Omega$. We show that there exists at least one nonconstant solution u_ε to this problem provided ε is small. Moreover, we prove that $u_\varepsilon \in L^\infty(\Omega)$ by using Moser-Nash iteration.

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