Differential
Equations
& Applications
Volume 10, Number 2 (2018), 147–161
doi:10.7153/dea-2018-10-09

EXISTENCE THEORY FOR NONLINEAR STURM–LIOUVILLE PROBLEMS WITH NON–LOCAL BOUNDARY CONDITIONS

DANIEL MARONCELLI AND JESÚS RODRÍGUEZ

Abstract. In this work we provide conditions for the existence of solutions to nonlinear Sturm-Liouville problems of the form

$$(p(t)x'(t))' + q(t)x(t) + \lambda x(t) = f(x(t))$$

subject to non-local boundary conditions

$$ax(0) + bx'(0) = \eta_1(x) \text{ and } cx(1) + dx'(1) = \eta_2(x).$$

Our approach will be topological, utilizing Schaefer's fixed point theorem and the Lyapunov-Schmidt procedure.

Mathematics subject classification (2010): 34B24.
Keywords and phrases: Existence theory, Sturm-Liouville problem, boundary conditions.

REFERENCES

[1] A. BOUCHERIF, *Second-order boundary value problems with integral boundary conditions*, Nonlinear Anal. **70** (2009), 364–371.
[2] X. CHEN AND Z. DU, *Existence of Positive Periodic Solutions for a neutral delay predator-prey model with Hassell-Varley type functional response and impulse*, Qual. Theory Dyn. Syst., (2017), doi:10.1007/s12346-017-0223-6.
[3] S. CHOW AND J. K. HALE, *Methods of Bifurcation Theory*, Springer, Berlin, 1982.
[4] P. DRÁBEK, *On the resonance problem with nonlinearity which has arbitrary linear growth*, J. Math. Anal. Appl. **127** (1987), 435–442.
[5] P. DRÁBEK, *Landesman-Lazer type condition and nonlinearities with linear growth*, Czechoslovak Math. J. **40** (1990), 70–86.
[6] P. DRÁBEK AND S. B. ROBINSON, *Resonance problems for the p-laplacian*, J. Funct. Anal. **169** (1999), 189–200.
[7] Z. I. DU AND J. YIN, *A Second Order Differential Equation with Generalized Sturm-Liouville Integral Boundary Conditions at Resonance*, Filomat **28** (2014), 1437–1444.
[8] R. IANNACCI AND M. N. NKASHAMA, *Nonlinear two point boundary value problems without Landesman-Lazer condition*, Proc. Amer. Math. Soc. **106** (1989), 943–952.
[9] W. G. KELLEY AND A. C. PETERSON, *The theory of differential Equations*, Springer, 2010.
[10] R. A. KHAN, *The generalized method of quasilinearization and nonlinear boundary value problems with integral boundary conditions*, Electronic. J. Qual. Theory Differ. Equ. **19** (2003), 1–15.
[11] E. M. LANDESMAN AND A. C. LAZER, *Nonlinear perturbations of linear elliptic boundary value problems at resonance*, J. Math. Mech. **19** (1969), 609–623.
[12] A. C. LAZER AND D. F. LEACH, *Bounded perturbations of forced harmonic oscillators at resonance*, Ann. Mat. Pure Appl. **82** (1969), 49–68.
[13] X. LIN AND Q. ZHANG, *Existence of solution for a p-Laplacian multi-point boundary value problem at resonance*, Qual. Theory Dyn. Syst., (2017), doi:10.1007/s12346-017-0259-7.
[14] R. MA, *Nonlinear discrete Sturm-Liouville problems at resonance*, Nonlinear Anal. **67** (2007), 3050–3057.

Paper DEA-10-09

[15] D. MARONCELLI AND J. RODRÍGUEZ, *Existence theory for nonlinear Sturm-Liouville problems with unbounded nonlinearities*, Differ. Equ. Appl. **6** (2014), 455–466.
[16] D. MARONCELLI AND J. RODRÍGUEZ, *On the solvability of nonlinear impulsive boundary value problems*, Topol. Methods Nonlinear Anal. **44** (2014), 381–398.
[17] H. PANG AND Y. TONG, *Symmetric positive solutions to a second-order boundary value problem with integral boundary conditions*, Bound. Value Probl. **150** (2013), 1–9.
[18] JESÚS F. RODRÍGUEZ, *Existence theory for nonlinear eigenvalue problems*, Appl. Anal. **87** (2008), 293–301.
[19] J. RODRÍGUEZ, *Nonlinear discrete Sturm-Liouville problems*, J. Math. Anal. Appl. **308** (2005), 380–391.
[20] J. RODRÍGUEZ AND A. SUAREZ, *Existence of solutions to nonlinear boundary value problems*, Differ. Equ. Appl. **9** (2017), 1–11.
[21] J. RODRÍGUEZ AND A. SUAREZ, *On nonlinear perturbations of Sturm-Liouville problems in discrete and continuous settings*, Differ. Equ. Appl. **8** (2016), 319–334.
[22] J. RODRÍGUEZ AND Z. ABERNATHY, *On the solvability of nonlinear Sturm-Liouville problems*, J. Math. Anal. Appl. **387** (2012), 310–319.
[23] X. M. ZHANG AND W. G. GE, *Positive solutions for a class of boundary-value problems with single integral boundary conditions*, Computers Mathematics with Applications **58** (2009), 203–215.