NONLOCAL BOUNDARY VALUE PROBLEMS
FOR (p,q)–DIFFERENCE EQUATIONS

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Abstract. In this paper we study existence and uniqueness of solutions for a boundary value problem for (p,q)-difference equations with nonlocal integral boundary conditions, by using classical fixed point theorems. Examples illustrating the main results are also presented.

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REFERENCES

[1] C. R. ADAMS, *On the linear ordinary q-difference equation*, Ann. of Math. **30** (1929), 195–205.

[2] B. AHMAD, S. K. NTOUYAS, *Boundary value problems for q-difference inclusions*, Abstr. Appl. Anal., vol. 2011, Article ID 292860, 15 pages.

[3] B. AHMAD, S. K. NTOUYAS, J. TARIBOON, *Quantum Calculus: New Concepts, Impulsive IVPs and BVPs, Inequalities*, Trends in Abstract and Applied Analysis, vol. 4, World Scientific, Singapore, 2016.

[4] D. W. BOYD, J. S. W. WONG, *On nonlinear contractions*, Proc. Amer. Math. Soc. **20** (1969), 458–464.

[5] I. BURBAN, *Two-parameter deformation of the oscillator algebra and (p,q)-analog of two-dimensional conformal field theory*, J. Nonlinear Math. Phys. (2013), 384–391.

[6] R. D. CARMICHAEL, *The general theory of linear q-difference equations*, Am. J. Math. **34** (1912), 147–168.

[7] R. CHAKRABARTI, R. JAGANNATHAN, *A (p,q)-oscillator realization of two-parameter quantum algebras*, J. Phys. A. Math. Gen. **24** (1991), L711-0L718.

[8] A. DOBROGOWSKA, A. ODZIJEWICZ, *Second order q-difference equations solvable by factorization method*, J. Comput. Appl. Math. **193** (2006), 319–346.

[9] M. EL-SHAHED, H. A. HASSAN, *Positive solutions of q-difference equation*, Proc. Amer. Math. Soc. **138** (2010), 1733–1738.

[10] T. ERNST, *The history of q-calculus and a new method*, UUDM Report 2000:16, Department of Mathematics, Uppsala University, 2000, ISSN:1101-3591.

[11] R. FERREIRA, *Nontrivial solutions for fractional q-difference boundary value problems*, Electron. J. Qual. Theory Differ. Equ., no. 70 (2010), pp. 1–10.

[12] G. GASPER, M. RAHMAN, *Basic Hypergeometric Series*, Cambridge University Press, Cambridge, 1990.

[13] A. GRANAS, J. DUGUNDJI, *Fixed Point Theory*, Springer-Verlag, New York, 2003.

[14] M. N. HOUNKONNOU, J. D. BUKWELI KYEMBA, *$R(p,q)$-calculus differentiation and integration*, SUT J. Math. **49**, no. 2, (2013), 145–167.

[15] H. F. JACKSON, *On q-definite integral*, Quart. J. Pure and Appl. Math. **41** (1910), 193–203.

[16] H. F. JACKSON, *q-Difference equations*, Am. J. Math. **32** (1910), 305–314.

[17] R. JAGANNATHAN, K. SRINIVASA RAO, *Two-parameter quantum algebras, twin-basic number, and associated generalized hypergeometric series*, `arX1v:math/0602613v1` [math.NT] 27 Feb 2006.

[18] V. KAC, P. CHEUNG, *Quantum Calculus*, Springer, New York, 2002.

[19] J. MA, J. YANG, *Existence of solutions for multi-point boundary value problem of fractional q-difference equation*, Electron. J. Qual. Theory Differ. Equ., no. 92 (2011), pp. 1–10.

[20] T.E. MASON, *On properties of the solution of linear q-difference equations with entire function coefficients*, Am. J. Math. **37** (1915), 439–444.

[21] M. MURSALEEN, K. J. ANSARI, A. KHAN, *On (p,q)-analogue of Bernstein Operators*, App. Math. Comp. **266** (2015), 874–882.

[22] V. SAHAI, S. YADAV, *Representations of two parameter quantum algebras and (p,q)-special functions*, J. Math. Anal. Appl. **335** (2007), 268–279.

[23] P. NJIONOU SADJANG, *On the fundamental theorem of (p,q)-calculus and some (p,q)-Taylor formulas*, `arXiv:1309.3934v1`.

[24] J. TARIBOON, S. K. NTOUYAS, *Quantum calculus on finite intervals and applications to impulsive difference equations*, Adv. Difference Equ. (2013) 2013:282.