

ON SIGN-CHANGING SOLUTIONS FOR RESONANT (p, q) -LAPLACE EQUATIONS

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Abstract. We provide two existence results for sign-changing solutions to the Dirichlet problem for the family of equations $-\Delta_p u - \Delta_q u = \alpha|u|^{p-2}u + \beta|u|^{q-2}u$, where $1 < q < p$ and α, β are parameters. First, we show the existence in the resonant case $\alpha \in \sigma(-\Delta_p)$ for sufficiently large β , thereby generalizing previously known results. The obtained solutions have negative energy. Second, we show the existence for any $\beta \geq \lambda_1(q)$ and sufficiently large α under an additional nonresonant assumption, where $\lambda_1(q)$ is the first eigenvalue of the q -Laplacian. The obtained solutions have positive energy.

Mathematics subject classification (2010): 35J62, 35J20, 35P30.

Keywords and phrases: (p, q) -Laplacian, generalized eigenvalue problem, nodal solutions, linking methods, indefinite nonlinearity.

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