

## LOWER BOUNDS FOR THE FIRST ZERO FOR NONLINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

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*Abstract.* We consider establishing lower bounds for the first zero of the solution of the nonlinear second order initial value problem

$$\begin{aligned}(p(x)y'(x))' + f(x, y(x)) &= 0, \quad x \geq 0 \\ y(0) = a > 0, \quad y'(0) &= 0.\end{aligned}$$

Using the linear case as a starting point, we prove several of these theorems, comparing them by considering several examples.

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### REFERENCES

- [1] PEDRO ALMENAR AND LUCAS JÓDAR, *An upper bound for the distance between zero and a critical point of a solution of a second order linear differential equation*, Computers and Mathematics with Applications **63**, (2012), 310–317.
- [2] R. C. BROWN AND D. B. HINTON, *Opial's inequality and oscillation of 2nd order equations*, Proceedings of the American Mathematical Society **125** (1997), 1123–1129.
- [3] B. J. HARRIS AND Q. KONG, *On the oscillation of differential equations with an oscillatory coefficient*, Transactions of the American Mathematical Society **347**, (1995), 1831–1839.
- [4] B. J. HARRIS, *On an inequality of Lyapunov for disfocality*, J. Math. Anal. Appl. **140**, (1990), 495–500.
- [5] MAN KAM KWONG, *On Lyapunov's inequality for disfocality*, Journal of Mathematical Analysis and Applications **83**, (1981), 486–494.
- [6] RICHARD R. MOORE, *The behavior of solutions of a linear differential equation of second order*, Pacific J. Math **5**, (1955), 126–145.
- [7] WILLIAM T. PATULA, *On the distance between zeros*, Proceedings of the American Mathematical Society **17**, (1975), 247–251.