

LYAPUNOV–TYPE INEQUALITIES FOR THIRD–ORDER LINEAR DIFFERENTIAL EQUATIONS UNDER THE NON–CONJUGATE BOUNDARY CONDITIONS

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Abstract. In this paper, we obtain the best constant in the Lyapunov-type inequality for third-order linear differential equations under the non-conjugate boundary conditions by bounding the Green function of the same problem. In this direction, to the best of our knowledge, there is no paper dealing with Lyapunov-type inequalities for the non-conjugate boundary value problems. By using such inequalities, we obtain sharp lower bounds for the eigenvalues of corresponding equations.

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