

EXISTENCE AND UNIQUENESS OF MONOTONE POSITIVE SOLUTIONS FOR A THIRD-ORDER THREE-POINT BOUNDARY VALUE PROBLEM

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Abstract. In this paper, we study the existence and uniqueness of monotone positive solutions for a class of nonlinear third-order three-point boundary value problem. The main tool is a fixed point theorem of generalized concave operators in ordered Banach spaces. An example is given to illustrate the main result.

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