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LYAPUNOV INEQUALITIES FOR TWO–PARAMETRIC QUANTUM HAMILTONIAN SYSTEMS AND THEIR APPLICATIONS

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Abstract. This paper deals with study of the two-parametric quantum Hamiltonian systems. The main objective in our study is Lyapunov inequalities of the two-parametric quantum Hamiltonian systems. In this paper, we first define two-parametric quantum analogous of the Leibniz rule, Cauchy-Schwarz and Holder inequalities and consequently as theoretical part of our main results, by the use of new Leibniz rule and Cauchy-Schwarz inequality on the considered Hamiltonian systems we obtain corresponding Lyapunov inequalities. Applicability of the obtained Lyapunov inequalities is examined by presenting a disconjugacy and at the same time a nonexistence criterion for the related Hamiltonian systems.

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