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EXISTENCE AND MULTIPLICITY SOLUTIONS FOR A NONLOCAL EQUATION OF KIRCHHOFF TYPE

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Abstract. In this paper, we study the nonlinear Kirchhoff equation

$$-\left(1+b\int_{\mathbb{R}^3}|\nabla u|^2dx\right)\Delta u+V(x)u=g(x,u)\quad \text{in } \mathbb{R}^3,$$

where the potential V and the primitive of g are allowed to be sign-changing and g is local superlinear. Under some assumptions on V and g, we get at least one nontrivial solution and infinitely many nontrivial solutions for this equation. Recent results in the literature are generalized and significantly improved.

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