A GRONWALL INEQUALITY AND THE CAUCHY–TYPE PROBLEM BY MEANS OF ψ–HILFER OPERATOR

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Abstract. In this paper, we propose a generalized Gronwall inequality through the fractional integral with respect to another function. The Cauchy-type problem for a nonlinear differential equation involving the ψ-Hilfer fractional derivative and the existence and uniqueness of solutions are discussed. Finally, through generalized Gronwall inequality, we prove the continuous dependence of data on the Cauchy-type problem.

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