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HÖLDER CONTINUITY UP TO THE BOUNDARY OF SOLUTIONS TO NONLINEAR FOURTH–ORDER ELLIPTIC EQUATIONS WITH NATURAL GROWTH TERMS

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Abstract. In a bounded open set $\Omega \subset \mathbb{R}^n$, $n \geqslant 3$, we consider the nonlinear fourth-order partial differential equation $\sum_{|\alpha|=1,2}(-1)^{|\alpha|}D^{\alpha}A_{\alpha}(x,u,Du,D^2u)+B(x,u,Du,D^2u)=0$. It is assumed that the principal coefficients $\{A_{\alpha}\}_{|\alpha|=1,2}$ satisfy the growth and coercivity conditions suitable for the energy space $\mathring{W}^{1,q}_{2,p}(\Omega)=\mathring{W}^{1,q}(\Omega)\cap\mathring{W}^{2,p}(\Omega)$, $1<p<n/2$, $2p<q<n$. The lower-order term $B(x,u,Du,D^2u)$ behaves as $b(u)\big\{|Du|^q+|D^2u|^p\big\}+g(x)$ where $g\in L^{\tau}(\Omega)$, $\tau>n/q$. We establish the Hölder continuity up to the boundary of any solution $u\in\mathring{W}^{1,q}_{2,p}(\Omega)\cap L^{\infty}(\Omega)$ by using the measure density condition on $\partial\Omega$, an interior local result and a modified Moser method with special test function.

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Paper DEA-11-03

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