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RAZUMIKHIN METHOD TO DELAY DIFFERENTIAL EQUATIONS WITH NON–INSTANTANEOUS IMPULSES

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Abstract. The stability for delay differential equations with non-instantaneous impulses is studied using Lyapunov like functions and the Razumikhin technique. In these differential equations we have impulses, which start abruptly at some points and their action continue on given finite intervals. Sufficient conditions are given and they use comparison results for nonlinear scalar non-instantaneous impulsive equation without any delay. Examples are given to illustrate our stability properties and the influence of non-instantaneous impulses on the behavior of the solution.

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