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GENERALIZED FIRST ORDER DYNAMIC EQUATIONS ON TIME SCALES WITH Δ–CARATHÉODORY FUNCTIONS

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Abstract. In this paper we consider a first order dynamic equation on time scales in which the right hand side is a Δ-Carathéodory function, which is not necessarily continuous. We generalize this discontinuous dynamic equation using Henstock–Kurzweil Δ-integral and establish results concerning existence of solutions using simple analysis. Uniqueness of solutions is obtained using an Osgood type condition. Moreover we introduce the concept of Henstock–Kurzweil Δ-equi-integrability and study continuous dependence and convergence of solutions.

Mathematics subject classification (2010): 26A39, 34A12, 34A26, 34N05.
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