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SECOND–ORDER ORDINARY DIFFERENTIAL EQUATIONS AND INCLUSIONS WITH A NEW KIND OF INTEGRAL AND MULTI–STRIP BOUNDARY CONDITIONS

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Abstract. In this paper, we study the existence of solutions for nonlinear second-order ordinary differential equations and inclusions with nonlinearity depending upon the unknown function together with its first derivative, supplemented with a new kind of integral and multi-strip boundary conditions. Krasnoselskii fixed point theorem and Banach contraction mapping principle are employed to prove the existence and uniqueness results for the single-valued boundary value problem. In the multi-valued case the nonlinear alternative of Leray-Schauder type is the key tool for studying convex valued right-hand side, while the case of non-convex valued right-hand side is handled with the aid of a fixed point theorem for contractive multivalued maps due to Covitz and Nadler. Examples are constructed for the illustration of the obtained results.

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