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STOKES AND NAVIER–STOKES PROBLEMS WITH NAVIER–TYPE BOUNDARY CONDITION IN L^P–SPACES

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Abstract. Using the semigroup theory for the Stokes equation with Navier type boundary conditions developed in [2, 3], we first prove the maximal L^p-L^q regularity for the strong, weak and very weak solutions of the inhomogeneous Stokes problem with Navier-type boundary conditions in a bounded domain Ω, not necessarily simply connected. We also prove the existence of a unique local in time classical solution to the Navier Stokes problem with Navier-type boundary conditions and show that it is global in time for small initial data.

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