

## SECOND ORDER TWO-PARAMETRIC QUANTUM BOUNDARY VALUE PROBLEMS

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**Abstract.** In this paper we study second order two-parametric quantum boundary value problems. The main aims of this paper are presented in two steps. In the first step, we consider second order two-parametric quantum boundary value problems with general nonlinearities and by the use of Krasnoselskii fixed point theorem on positive cones we provide some sufficient conditions to reach the existence, multiplicity and nonexistence of positive solutions. At the end of this step, some illustrative examples are given to show practical implementability of the obtained theoretical results. In the second step, we consider the corresponding two-parametric quantum eigenvalue problems and in the light of Lyapunov inequalities, we present a lower bound estimation for positive eigenvalues. We complete this step with a numerical evaluation to identify validity of the obtained lower bound.

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