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PROGRESSIVE CONTRACTIONS, MEASURES OF NON–COMPACTNESS AND QUADRATIC INTEGRAL EQUATIONS

T. A. BURTON AND I. K. PURNARAS

Abstract. Classical fixed point theorems often begin with the assumption that we have a mapping P of a non-empty, closed, bounded, convex set G in a Banach space into itself. Then a number of conditions are added which will ensure that there is at least one fixed point in the set G. These fixed point theorems have been very effective with many problems in applied mathematics, particularly for integral equations containing a term

$$\int_0^t A(t-s)v(t,s,x(s))ds,$$

because such terms frequently map sets of bounded continuous functions into compact sets. But there is a large and important class of integral equations from applied mathematics containing such a term with a coefficient function $f(t,x)$ which destroys all compactness. Investigators have then turned to Darbo's fixed point theorem and measures of non-compactness to get a (possibly non-unique) fixed point. In this paper:

a) We offer an elementary alternative to measures of non-compactness and Darbo's theorem by using progressive contractions. This method yields a unique fixed point (unlike Darbo's theorem) which, in turn, by default yields asymptotic stability as introduced in [1].

b) We lift the growth requirements in both x and t seen using Darbo's theorem.

c) We offer a technique for finding the mapping set G.

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