PROGRESSIVE CONTRACTIONS, MEASURES OF NON–COMPACTNESS AND QUADRATIC INTEGRAL EQUATIONS

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Abstract. Classical fixed point theorems often begin with the assumption that we have a mapping \( P \) of a non-empty, closed, bounded, convex set \( G \) in a Banach space into itself. Then a number of conditions are added which will ensure that there is at least one fixed point in the set \( G \). These fixed point theorems have been very effective with many problems in applied mathematics, particularly for integral equations containing a term

\[
\int_0^t A(t-s)v(t,s,x(s))ds,
\]

because such terms frequently map sets of bounded continuous functions into compact sets. But there is a large and important class of integral equations from applied mathematics containing such a term with a coefficient function \( f(t,x) \) which destroys all compactness. Investigators have then turned to Darbo’s fixed point theorem and measures of non-compactness to get a (possibly non-unique) fixed point. In this paper:

a) We offer an elementary alternative to measures of non-compactness and Darbo’s theorem by using progressive contractions. This method yields a unique fixed point (unlike Darbo’s theorem) which, in turn, by default yields asymptotic stability as introduced in [1].

b) We lift the growth requirements in both \( x \) and \( t \) seen using Darbo’s theorem.

c) We offer a technique for finding the mapping set \( G \).


Keywords and phrases: Quadratic integral equations, progressive contractions, measures of non-compactness, unique fixed points.

REFERENCES