

POSITIVE SOLUTIONS TO A NONLINEAR SIXTH ORDER BOUNDARY VALUE PROBLEM

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Abstract. We consider a sixth order two point boundary value problem. Upper and lower estimates for positive solutions of the problem are proved. Sufficient conditions for the existence and nonexistence of positive solutions for the problem are obtained. An example is included to illustrate the results.

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