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ON THE SOLUTIONS FOR AN EXTENSIBLE BEAM EQUATION WITH INTERNAL DAMPING AND SOURCE TERMS

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Abstract. In this manuscript, we consider the nonlinear beam equation with internal damping and source term

$$u_{tt}+\Delta^2u+M(|\nabla u|^2)(-\Delta u)+u_t=|u|^{r-1}u$$

where $r>1$ is a constant, $M(s)$ is a continuous function on $[0,+\infty)$. The global solutions are constructed by using the Faedo-Galerkin approximations, taking into account that the initial data is in appropriate set of stability created from the Nehari manifold. The asymptotic behavior is obtained by the Nakao method.

Mathematics subject classification (2010): 35B40, 35G30, 35K30, 74K20, 74K10, 74Kxx, 93D15, 93D20.

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