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SOME PROPERTIES ABOUT COMPLEX DIFFERENCE EQUATIONS OF MALMQUIST TYPE

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Abstract. This article presents versions of the Malmquist type equation. We study the growth of transcendental meromorphic solutions of some complex $(qz+c)$ difference equations and find lower bounds for Nevanlinna lower order for meromorphic solutions of such equations. We also obtain a $(qz+c)$ difference version of Tumura-Clunie theorem, which improves the results of Zheng and Chen[25].

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