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ASYMPTOTICALLY SELF–SIMILAR GLOBAL SOLUTIONS FOR HARDY–HÉNON PARABOLIC SYSTEMS

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Abstract. In this paper we study the nonlinear parabolic system $\partial_t u = \Delta u + a|x|^{-\gamma}|v|^{p-1}v$, $\partial_t v = \Delta v + b|x|^{-\rho}|u|^{q-1}u$, $t>0$, $x\in\mathbb{R}^N\backslash\{0\}$, $N\geqslant 1$, $a,b\in\mathbb{R}$, $0\leqslant\gamma<\min(N,2)$, $0<\rho<\min(N,2)$, $p,q>1$. Under conditions on the parameters p,q,γ and ρ we show the existence and uniqueness of global solutions for initial values small with respect of some norms. In particular, we show the existence of self-similar solutions with initial value $\Phi=(\varphi_1,\varphi_2)$, where φ_1, φ_2 are homogeneous initial data. We also prove that some global solutions are asymptotic for large time to self-similar solutions.

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REFERENCES

[1] B. BEN SLIMENE, S. TAYACHI AND F.B.WEISSLER, *Well-posedness, global existence and large time behavior for Hardy-Hénon parabolic equations*, Nonlinear Anal., **152** (2017), pp. 116–148.
[2] H. BREZIS AND T. CAZENAVE , *A nonlinear heat equation with singular initial data*, J. Analyse Math., **68** (1996), pp. 277–304.
[3] M. CANNONE, *A generalization of a theorem by Kato on Navier-Stokes equations*, Rev. Math. Ib., **13** (1997), pp. 515–541.
[4] M. CANNONE, Y. Meyer and F. Planchon, *Solutions autosimilaires des équations de Navier-Stokes*, Séminaire "Équations aux Dérivées Partielle" de l'école polytechnique, Exposé VIII, 1993-1994.
[5] T. CAZENAVE AND F. B. WEISSLER, *Asymptotically self-similar global solutions of the nonlinear Shrödinger and heat equations*, Math. Z., **228** (1998), pp. 83–120.
[6] T. CAZENAVE, F. DICKESTEIN AND F. B. WEISSLER, *An equation whose Fujita critical exponent is not given by scaling*, Nonlinear Anal., **68** (2008), pp. 862–874.
[7] Y. GIGA AND T. MIYAKAWA, *Navier-Stokes flow in $\mathbb{R}^3$ with measures as initial vorticity and Morrey spaces*, Comm. in PDE., **14** (1989), pp. 577–618.
[8] T. KATO, *Strong L^p-solutions of the Navier-Stokes equation in $\mathbb{R}^m$, with applications to weak solutions*, Math. Z., **187** (1984), pp. 471–480.
[9] H. FUJITA AND T. KATO, *On the Navier-Stokes initial value problem* , Arch. Rational Mech. Anal., **16** (1964), pp. 269–315.
[10] H. FUJITA AND T. KATO, *On the non stationary Navier-Stokes system*, Rend. Sem. Math. Univ. Padova, **32** (1962), pp. 243–260.
[11] Q. H. PHAN, *Singularity and blow-up estimates via Liouville-type theorems for Hardy-Hénon parabolic equations*, J. Evol. Equ., **13** (2013), pp. 411–442.
[12] Q. H. PHAN AND PH. SOUPLET, *Liouville-type theorems and bounds of solutions of Hardy-Hénon equations*, (English) [J] J. Differential Equations, **252** (2012), pp. 2544–2562 .
[13] S. SNOUSSI and S. TAYACHI, *Asymptotic self-similar behavior of solutions for a seminlinear parabolic system*, Comm. Cont. Math., **3** (2001), pp. 363–392.
[14] S. SNOUSSI, S. TAYACHI AND F. B. WEISSLER, *Asymptotically self-similar global solutions of a semilinear parabolic equation with a nonlinear gradient term*, Proc. Royal Soc. Edinburgh Sect. A, **129** (1999), pp. 1291–1307.

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[15] S. SNOUSSI, S. TAYACHI AND F. B. WEISSLER, *Asymptotically self-similar global solutions of a general semilinear heat equation*, Math. Ann., **321** (2001), pp. 131–155.
[16] S. SNOUSSI AND S. TAYACHI, *Global existence, asymptotic behavior and self-similar solutions for a class of semilinear parabolic systems*, Nonlinear Anal., **48** (2002), pp. 13–53.
[17] X. WANG, *On the Cauchy problem for reaction-diffusion equations*, Trans. Amer. Math. Soc., **337** (1993), pp. 549–590.
[18] Y. YAMAUCHI, *Blow-up results for a reaction-diffusion system*, Methods Appl. Anal., **13** (2006), pp. 337–350.