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EIGENVALUE CRITERIA FOR EXISTENCE AND NONEXISTENCE OF POSITIVE SOLUTIONS FOR $\alpha-$ORDER FRACTIONAL DIFFERENTIAL EQUATIONS, $(2<\alpha<3)$, ON THE HALF–LINE

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Abstract. This article concerns nonexistence and existence of positive solutions to the fractional differential equation

$$\begin{cases} D^{\alpha}u(t)+f(t,u(t))=0, \quad 0\leqslant t<\infty, \\ u(0)=D^{\alpha-2}u(0)=\lim_{t\to\infty}D^{\alpha-1}u(t)=0, \end{cases}$$

where $\alpha\in(2,3)$, D^{α} is the standard Riemann-Liouville derivative and $f:\mathbb{R}^{+}\times\mathbb{R}^{+}\to\mathbb{R}^{+}$ is a continuous function. The main results obtained here, are under eigenvalue criteria.

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