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FINAL STATE PROBLEM FOR THE NONLOCAL NONLINEAR SCHRÖDINGER EQUATION WITH DISSIPATIVE NONLINEARITY

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Abstract. We consider the asymptotic behavior of solutions to the nonlocal nonlinear Schrödinger equation with dissipative nonlinearity. We prove that there exists a solution which has different behavior from that of the typical cubic nonlinear Schrödinger equation.

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