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POSITIVE SOLUTIONS FOR A SINGULAR COUPLED SYSTEM OF NONLINEAR HIGHER–ORDER FRACTIONAL q–DIFFERENCE BOUNDARY VALUE PROBLEMS WITH TWO PARAMETERS

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Abstract. In this paper, we are concern with the existence of positive solutions for a singular system of nonlinear fractional q-difference equations with coupled integral boundary conditions and two parameters. By using the properties of the Green's function and Guo-Krasnosel'skii fixed point theorem, some existence results of at least one positive solution are obtained. As applications, two examples are presented to illustrate the main results.

Mathematics subject classification (2010): 39A13, 34B18, 34A08.

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Paper DEA-11-25

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