

ANTISYMMETRIC SOLUTIONS FOR A CLASS GENERALIZED QUASILINEAR SCHRÖDINGER EQUATIONS

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Abstract. In this paper we consider the existence of antisymmetric solutions for the generalized quasilinear Schrödinger equation in $H^1(\mathbb{R}^N)$:

$$-\operatorname{div}(\vartheta(u)\nabla u) + \frac{1}{2}\vartheta(u)|\nabla u|^2 + V(x)u = f(u) \text{ in } \mathbb{R}^N,$$

where $N \geq 3$, $V(x)$ is a positive continuous potential, $f(u)$ is of subcritical growth and $\vartheta: \mathbb{R} \rightarrow [1, +\infty)$ is an even C^1 -function satisfying some suitable hypotheses. By considering a minimizing problem restricted on a partial Nehari manifold, we prove the existence of antisymmetric solutions via deformation lemma.

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