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ASYMPTOTICS FOR THE SOBOLEV TYPE EQUATIONS WITH PUMPING

JHON J. PÉREZ

Abstract. We consider the large time asymptotic behavior of solutions to the initial-boundary value problem

$$\begin{cases} \partial_t(u-u_{xx})+(1+t)^n u\,u_x-u_{xx}=0, x\in\mathbb{R}, t>0, \\ u(0,x)=u_0(x), x\in\mathbb{R}, \\ u(t,x)\to a_\pm, x\to\pm\infty, t>0, \end{cases}$$

where $n\in\mathbb{N}$. We find large time asymptotic formulas of solutions for three different cases 1) $a_\pm=\pm 1$, 2) $a_\pm=\mp 1$, 3) $a_\pm=0$.

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