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MULTIPLICITY OF SOLUTIONS FOR A FRACTIONAL $p-$KIRCHHOFF TYPE PROBLEM WITH SIGN–CHANGING WEIGHTS FUNCTION

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Abstract. In this paper, we consider the existence of multiple solutions for the following fractional $p-$Kirchhoff type problem

$$\begin{cases} \left(\iint_{\mathbb{R}^{2n}} \frac{|u(x)-u(y)|^p}{|x-y|^{n+ps}} dx\,dy\right)^{\theta/p} (-\Delta)_p^s u = f(x)|u|^{q-1}u + g(x)|u|^{r-1}u, & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^n \backslash \Omega, \end{cases} \tag{0.1}$$

where Ω is an open bounded set in $\mathbb{R}^n$, $p>1$, $\theta \geqslant 0$, $0<q<\theta+p-1<r<p_s^*-1$ with $p_s^* = \frac{np}{n-ps}$ for $n>ps$ and $s\in(0,1)$ fixed, $f(x)$ and $g(x)$ are sign-changing continuous functions in Ω, $(-\triangle)_p^s u$ denotes the fractional $p-$Laplacian operator. We obtain the multiplicity of solutions to (0.1) by using fibering map analysis and the Nehari manifold approach.

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