

ON WEAKLY NONLINEAR BOUNDARY VALUE PROBLEMS ON INFINITE INTERVALS

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Abstract. In this paper, we study weakly nonlinear boundary value problems on infinite intervals. For such problems, we provide criteria for the existence of solutions as well as a qualitative description of the behavior of solutions depending on a parameter. We investigate the relationship between solutions to these weakly nonlinear problems and the solutions to a set of corresponding linear problems.

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