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ANALYTICAL APPROXIMATION OF TIME–FRACTIONAL TELEGRAPH EQUATION WITH RIESZ SPACE–FRACTIONAL DERIVATIVE

SAFIYEH MOHAMMADIAN, YAGHOUB MAHMOUDI AND FARHAD DASTMALCHI SAEI

Abstract. In this study, fractional reduced differential transform method (FRDTM) is developed to derive a semi- analytical solution of fractional partial differential equations which involves Riesz space fractional derivatives. We focus primarily on implementing the novel algorithm of FRDTM to Riesz space -fractional telegraph equation while the telegraph equation has fractional order. Some theorems with their proofs which are used for calculating differential transform of Riesz derivative operator are presented, as well as the convergence condition and the error bound of the proposed method are established. To illustrate the reliability and capability of the method, some examples are provided. The results reveal that the algorithm is very effective and uncomplicated.

Mathematics subject classification (2010): 65Z05, 35Q60, 35Q99.
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