

EXISTENCE OF POSITIVE SOLUTION FOR A CLASS OF NONLOCAL PROBLEM WITH STRONG SINGULARITY AND LINEAR TERM

AI-JUN HOU AND JIA-FENG LIAO

Abstract. We consider a class of nonlocal problem with strong singularity and linear term. Combining with the variational method and Nehari manifold, a necessary and sufficient condition that shows the existence of positive solution is obtained.

Mathematics subject classification (2010): 35B09, 35D30, 35J75.

Keywords and phrases: Nonlocal problem, strong singularity, linear term, variational method, Nehari manifold.

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