Differential
Equations
& Applications
Volume 12, Number 3 (2020), 291–312
doi:10.7153/dea-2020-12-19

EXISTENCE AND UNIQUENESS OF SOLUTIONS TO THIRD–ORDER BOUNDARY VALUE PROBLEMS: ANALYSIS IN CLOSED AND BOUNDED SETS

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Abstract. The aim of this work is to develop a fuller theory regarding the existence, uniqueness and approximation of solutions to third-order boundary value problems via fixed point methods. To develop this deeper understanding of qualitative properties of solutions, our strategy involves an analysis of the problem under consideration, and its associated operator equations, within closed and bounded sets. This enables our new results to apply to a wider range of problems than those covered in the recent literature and we discuss several examples to illustrate the nature of these advancements.

Mathematics subject classification (2010): 34B15.

Keywords and phrases: Third-order boundary value problem, existence, uniqueness and approximation of solutions, contraction mappings, fixed point theorems, closed and bounded sets, two and three-point boundary conditions.

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