EXISTENCE AND UNIQUENESS FOR FRACTIONAL ORDER
FUNCTIONAL DIFFERENTIAL EQUATIONS WITH HILFER DERIVATIVE

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Abstract. We investigate fractional order delay and neutral differential equations. By using Banach fixed point theorem we establish existence and uniqueness of the solutions for fractional order functional differential equations involving Hilfer fractional derivative in the weighted spaces.

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