COUPLED HILFER AND HADAMARD RANDOM FRACTIONAL DIFFERENTIAL SYSTEMS WITH FINITE DELAY IN GENERALIZED BANACH SPACES

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Abstract. This article deals with some questions of existence and uniqueness of random solutions for some coupled systems of random Hilfer and Hilfer–Hadamard fractional differential equations with finite delay. We use some generalizations of classical random fixed point theorems on generalized Banach spaces.

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REFERENCES

[1] S. ABBAS, W. ALBARAKATI, M. BENCHOHRA AND Y. ZHOU, *Weak solutions for partial pettis Hadamard fractional integral equations with random effects*, J. Int. Equ. Appl. **29** (4) (2017) 1–19.

[2] S. ABBAS, M. BENCHOHRA, J. R. GRAEF AND J. HENDERSON, *Implicit Fractional Differential and Integral Equations: Existence and Stability*, De Gruyter, Berlin, 2018.

[3] S. ABBAS, M. BENCHOHRA, J. E. LAZREG AND Y. ZHOU, *A Survey on Hadamard and Hilfer fractional differential equations: Analysis and Stability*, Chaos, Solitons Fractals, 102 (2017) 47–71.

[4] S. ABBAS, M. BENCHOHRA AND G. M. N'GUÉRÉKATA, *Topics in Fractional Differential Equations*, Springer, New York, 2012.

[5] S. ABBAS, M. BENCHOHRA AND G. M. N'GUÉRÉKATA, *Advanced Fractional Differential and Integral Equations*, Nova Science Publishers, New York, 2015.

[6] S. ABBAS, M. BENCHOHRA AND Y. ZHOU, *Coupled Hilfer fractional differential systems with random effects*, Adv. Difference Equ. **2018**: 369, 12 pp.

[7] G. ALLAIRE AND S. M. KABER, *Numerical Linear Algebra*, ser. Texts in Applied Mathematics, Springer, New York, 2008.

[8] D. BOTHE, *Multivalued perturbation of m-accretive differential inclusions*, Isr. J. Math. **108** (1998), 109–138.

[9] H. W. ENGL, *A general stochastic fixed-point theorem for continuous random operators on stochastic domains*, J. Math. Anal. Appl. **66** (1978), 220–231.

[10] K. M. FURATI AND M. D. KASSIM, *Non-existence of global solutions for a differential equation involving Hilfer fractional derivative*, Electron. J. Differential Equations 2013, No. 235, 10 pp.

[11] K. M. FURATI, M. D. KASSIM, AND N.-E. TATAR, *Existence and uniqueness for a problem involving Hilfer fractional derivative*, Comput. Math. Appl. **64** (2012), 1616–1626.

[12] J. R. GRAEF, J. HENDERSON, A. OUAHAB, *Some Krasnosel'skii type random fixed point theorems*, J. Nonlinear Funct. Anal. **2017** (2017), 1–34, Article ID 46.

[13] R. HILFER, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.

[14] S. ITOH, *Random fixed point theorems with applications to random differential equations in Banach spaces*, J. Math. Anal. Appl. **67** (1979), 261–273.

[15] R. KAMOCKI AND C. OBCZŃSKI, *On fractional Cauchy-type problems containing Hilfer's derivative*, Electron. J. Qual. Theory Differ. Equ., 2016, No. 50, 1–12.

[16] A. A. KILBAS, *Hadamard-type fractional calculus*, J. Korean Math. Soc. **38** (6) (2001) 1191–1204.

[17] A. A. KILBAS, H. M. SRIVASTAVA AND J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, Elsevier Science B. V., Amsterdam, 2006.

[18] L. LIU, F. GUO, C. WU AND Y. WU, *Existence theorems of global solutions for nonlinear Volterra type integral equations in Banach spaces*, J. Math. Anal. Appl. **309** (2005), 638–649.

[19] K. LIU, J. R. WANG AND D. O'REGAN, *Ulam-Hyers-Mittag-Leffler stability for ψ-Hilfer delay differential equations*, Adv. Difference Equ. **2019**: 50, 12 pp.

[20] I. R. PETRE AND A. PETRUSEL, *Krasnoselskii's theorem in generalized Banach spaces and applications*, Electron. J. Qual. Theory Differ. Equ. (2012), No. 85, 20 pp.

[21] M. D. QASSIM, K. M. FURATI, AND N.-E. TATAR, *On a differential equation involving Hilfer-Hadamard fractional derivative*, Abstract Appl. Anal., vol. 2012, Article ID 391062, 17 pages, 2012.

[22] M. D. QASSIM AND N.-E. TATAR, *Well-posedness and stability for a differential problem with Hilfer-Hadamard fractional derivative*, Abst. Appl. Anal. Volume 2013, Article ID 605029, 12 pages, 2013.

[23] S. G. SAMKO, A. A. KILBAS AND O. I. MARICHEV, *Fractional Integrals and Derivatives. Theory and Applications*, Gordon and Breach, Amsterdam, 1987, Engl. Trans. from the Russian.

[24] M. L. SINACER, J. J. NIETO AND A. OUAHAB, *Random fixed point theorems in generalized Banach spaces and applications*, Random Oper. Stoch. Equ. **24** (2016), 93–112.

[25] V. E. TARASOV, *Fractional Dynamics: Application of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer, Heidelberg; Higher Education Press, Beijing, 2010.

[26] Ž. TOMOVSKI, R. HILFER AND H. M. SRIVASTAVA, *Fractional and operational calculus with generalized fractional derivative operators and Mittag-Leffler type functions*, Integral Transform. Spec. Funct. **21** (11) (2010), 797–814.

[27] R. S. VARGA, *Matrix iterative analysis*, Second revised and expanded edition, Springer Series in Computational Mathematics, 27, Springer-Verlag, Berlin, 2000.

[28] J.-R. WANG, AND Y. ZHANG, *Nonlocal initial value problems for differential equations with Hilfer fractional derivative*, Appl. Math. Comput. **266** (2015), 850–859.

[29] Y. ZHOU, J.-R. WANG, L. ZHANG, *Basic Theory of Fractional Differential Equations*, Second edition, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017.

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