Differential
Equations
& Applications
Volume 12, Number 4 (2020), 355–375
doi:10.7153/dea-2020-12-23

A VARIATIONAL METHOD FOR SOLVING QUASILINEAR ELLIPTIC SYSTEMS INVOLVING SYMMETRIC MULTI–POLAR POTENTIALS

ALI JABAR RASHIDI, MOHSEN SHEKARBAIGI

Abstract. In this paper, a system of quasilinear elliptic equations is investigated, which involves multiple critical Hardy-Sobolev exponents and symmetric multi-polar potentials. By employing the variational methods and analytic techniques, the relevant best constants are studied and the existence of $(\mathbb{Z}_k \times \mathbb{SO}(N-2))^2$-invariant solutions to the system is established.

Mathematics subject classification (2010): 35A15, 35B33, 35J70.
Keywords and phrases: Variational method, quasilinear elliptic system, critical Hardy-Sobolev exponent.

REFERENCES

[1] B. ABDELLAOUI, V. FELLI, I. PERAL, *Existence and nonexistence for quasilinear equations involving the p-Laplacian*, Boll. Unione Mat. Ital. Sez. B. **9**, (2006), 445–484.
[2] H. ATTOUCH, G. BUTTAZZO, G. MICHAILLE, *Variational analysis in sobolev and BV spaces*, The Society for Industrial and Applied Mathematics, 3600 University City Science Center, Philadelphia, PA. 19104–2688.
[3] L. CAFFARELLI, R. KOHN, L. NIRENBERG, *First order interpolation inequality with weights*, Compos. Math. **53**, (1984), 259–275.
[4] D. CAO, P. HAN, *Solutions to critical elliptic equations with multi-singular inverse square potentials*, J. Differential Equations. **224**, (2006), 332–372.
[5] D. CAO, S. PENG, *A note on the sign-changing solutions to elliptic problems with critical Sobolev and Hardy terms*, J. Differential Equations. **177**, (2001), 494–522.
[6] F. CATRINA, Z. WANG, *On the Caffarelli-Kohn-Nirenberg inequalities: Sharp constants, existence (and nonexistence) and symmetry of external functions*, Comm. Pure Appl. Math. **54**, (2001), 229–257.
[7] K. CHOU, C. CHU, *On the best constant for a weighted Sobolev-Hardy inequality*, J. London Math. Soc. **333**, (2007), 889–903.
[8] Y. DENG, Q. GAO, D. ZHANG, *Nodal solutions for Laplace equations with critical Sobolev and Hardy exponents on* $\mathbb{R}^N$, Discrete Contin. Dyn. Syst. **14**, (2006), 707–719.
[9] V. FELLI, S. TERRACINI, *Elliptic equations with multi-singular inverse-square potentials and critical nonlinearity*, Comm. Partial Differential Equations. **31**, (2006), 469–495.
[10] A. FERRERO, F. GAZZOLA, *Existence of solutions for singular critical growth semilinear elliptic equations*, J. Differential Equations. **177**, (2001), 494–522.
[11] J. P. GARCIA AZORERO, I. PERAL ALONSO, *Multiplicity of solutions for elliptic problems with critical exponent or with a nonsymmetric term*, Trans. Amer. Math. Soc. **323:2**, (1991), 877–895.
[12] N. GHOUSSOUB, C. YUAN, *Multiple solutions for quasilinear PDEs involving the critical Sobolev and Hardy exponents*, Trans. Amer. Math. Soc. **352**, (2000), 5703–5743.
[13] G. HARDY, J. LITTLEWOOD, G. PÓLYA, Inequalities, Cambridge University Press, Cambridge, UK, (1934).
[14] P. HAN, *Quasilinear elliptic problems with critical exponents and Hardy terms*, Nonlinear Anal. **61**, (2005), 735–758.
[15] Y. HUANG, D. KANG, *On the singular elliptic systems involving multiple critical Sobolev exponents*, Nonlinear Anal. **74**, (2011), 400–412.

Paper DEA-12-23

[16] E. JANNELLI, *The role played by space dimension in elliptic critical problems*, J. Differential Equations. **156**, (1999), 407–426.

[17] D. KANG, *On the quasilinear elliptic problems with critical Sobolev-Hardy exponents and Hardy terms*, Nonlinear Anal. **68**, (2008), 1973–1985.

[18] D. KANG, *Quasilinear elliptic problems involving multiple critical Hardy-Sobolev exponents*, Computers and Mathematics with Applications **57**, (2009), 107–116.

[19] D. KANG, *Concentration compactness principles for the systems of critical elliptic equations*, Differ Equ Appls. **4**, (2012), 435–444.

[20] P. L. LIONS, *The concentration compactness principle in the calculus of variations, the limit case (I)*, Rev Mat Iberoamericana. **1**, (1985), 145–201.

[21] P. L. LIONS, *The concentration compactness principle in the calculus of variations, the limit case (II)*, Rev Mat Iberoamericana. **1**, (1985), 45–121.

[22] R. PALAIS, *The principle of symmetric criticality*, Comm Math Phys. **69**, (1979), 19–30.

[23] D. SMETS, *Nonlinear Schrödinger equations with Hardy potential and critical nonlinearities*, Trans. Amer. Math. Soc. **357**, (2005), 2909–2938.

[24] S. TERRACINI, *On positive solutions to a class of equations with a singular coefficient and critical exponent*, Adv. Differential Equations. **2**, (1996), 241–264.

[25] J. VAZQUEZ, *A strong maximum principle for some quasilinear elliptic equations*, Appl Math Optim. **12**, (1984), 191–202.

[26] M. WILLEM, *Analyse fonctionnelle élémentaire*, Paris: Cassini Éditeurs. (2003).

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com