ON SINGULAR ELLIPTIC EQUATION WITH
SINGULAR NONLINEARITIES, HARDY–SOBOLEV
CRITICAL EXPONENT AND WEIGHTS

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Abstract. This article is devoted to the existence and multiplicity to the following singular elliptic equation with singular nonlinearities, Hardy-Sobolev critical exponent and weights:

$$\begin{cases} -\Delta u - \mu \frac{u}{|x|^2} = \frac{|u|^{p-2}u}{|x|^s} + \lambda \frac{u}{|x|^\alpha} |u|^{-\beta}, & x \in \Omega, \\ u > 0 & x \in \Omega, \\ u = 0 & x \in \partial\Omega. \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geqslant 3)$, $0 \in \Omega$, $\lambda > 0$, $0 \leqslant \mu < \overline{\mu}_N := (N-2)^2/4$, $p = 2^*(s) = 2(N-s)/(N-2)$ with $0 < s < 2$ is the critical Hardy-Sobolev critical exponent, $0 \leqslant \alpha < N(p-1+\beta)/p$, $0 < \beta < 1$ and $2 < p \leqslant 2^* := 2N/(N-2)$ is the critical Sobolev exponent.

By using the Nehari manifold and mountain pass theorem, the existence of at least four distinct solutions is obtained.

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