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PARABOLIC ANISOTROPIC PROBLEMS WITH LOWER ORDER TERMS AND INTEGRABLE DATA

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Abstract. In this paper we are concerned with the study of a class of second-order quasilinear parabolic equations involving Leray-Lions type operators with anisotropic growth conditions. By an approximation argument, we estabilsh the existence of entropy solutions in the framework of anisotropic parabolic Sobolev spaces when the initial condition and the data are assumed to be merely integrable. In addition, we prove that entropy solutions coincide with the renormalized solutions.

Mathematics subject classification (2010): 35K55, 35K58, 35K59, 46E35.
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