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A REMARK ON THE LOCAL WELL–POSEDNESS FOR A COUPLED SYSTEM OF MKDV TYPE EQUATIONS IN $H^s \times H^k$

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Abstract. We consider the initial value problem associated to a system consisting modified Korteweg-de Vries type equations

$$\begin{cases} \partial_t v + \partial_x^3 v + \partial_x(vw^2) = 0, & v(x,0) = \phi(x), \\ \partial_t w + \alpha\partial_x^3 w + \partial_x(v^2 w) = 0, & w(x,0) = \psi(x), \end{cases}$$

and using only bilinear estimates of the type $\|J^\gamma F^1_{b_1} \cdot J^\beta F^2_{b_2}\|_{L^2_x L^2_t}$, where J is the Bessel potential and $F^j_{b_j}$, $j = 1,2$ are multiplication operators, we prove the local well-posedness results for given data in low regularity Sobolev spaces $H^s(\mathbb{R}) \times H^k(\mathbb{R})$ for $\alpha \neq 0,1$. In this work we improve the previous result in [6], extending the LWP region from $|s-k| < 1/2$ to $|s-k| < 1$. This result is sharp in the region of the LWP with $s \leqslant 0$ and $k \leqslant 0$, in the sense of the trilinear estimates fails to hold.

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