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CAPUTO TYPE MODIFICATION OF THE ERDÉLYI–KOBER COUPLED IMPLICIT FRACTIONAL DIFFERENTIAL SYSTEMS WITH RETARDATION AND ANTICIPATION

MOKHTAR BOUMAAZA, MOUFFAK BENCHOHRA* AND JUAN J. NIETO

Abstract. In this paper, we deal with the existence and uniqueness of solutions of a coupled system of nonlinear implicit fractional differential equations of Caputo-type modification of the Erdélyi-Kober involving both retarded and advanced arguments. The arguments are based upon the Banach contraction principle and Schauder's fixed point theorem. An example is included to show the applicability of our outcomes.

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