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MULTIPLICITY RESULTS FOR CRITICAL FRACTIONAL EQUATIONS WITH SIGN–CHANGING WEIGHT FUNCTIONS

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Abstract. In this paper, we consider a time-independent fractional equation:

$$\begin{cases} (-\Delta)^s u = f(x)|u|^{2_s^*-2}u + g(x)|u|^{q-1}u, & x \in \Omega; \\ u = 0, & x \in R^N \setminus \Omega, \end{cases}$$

where Ω is a smooth bounded domain, $s \in (0,1)$, $N > 2s$, $0 < q < 1$, the coefficient functions f and g may change sign. We first obtain the existence of ground state solution by the Nehari method under the combined effect of coefficient functions. Then we find the multiplicity of positive solutions by Mountain pass theorem under some stronger conditions, and one of them is a ground state solution.

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