

EXISTENCE OF SOLUTIONS FOR A COUPLED SYSTEM OF CAPUTO TYPE FRACTIONAL-ORDER DIFFERENTIAL INCLUSIONS WITH NON-SEPARATED BOUNDARY CONDITIONS ON MULTIVALUED MAPS

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Abstract. Sufficient conditions for the existence of solutions to a coupled system of fractional-order differential inclusions associated with fractional non-separated boundary conditions for multivalued maps are established, by employing the nonlinear alternative of Leray–Schauder type. We emphasize that the methods of fixed point theory used in our analysis are standard, although their application to a system of fractional-order differential inclusions is new.

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REFERENCES

- [1] S. ABBAS, M. BENCHOHRA, J. R. GRAEF, J. HENDERSON, *Implicit fractional differential and integral equations: existence and stability*, **26**, Walter de Gruyter GmbH and Co KG, 2018.
- [2] S. ADLY, T. HADDAD, L. THIBAUT, *Convex sweeping process in the framework of measure differential inclusions and evolution variational inequalities*, *Math. Program. Ser. B*, **148** (2014), 5–47.
- [3] B. AHMAD, S. K. NTOUYAS, *Existence results for a coupled system of Caputo type sequential fractional differential equations with nonlocal integral boundary conditions*, *Appl. Math. Comput.*, **266** (2015), 615–622.
- [4] R. P. AGARWAL, B. AHMAD, D. GAROUT, A. ALSAEDI, *Existence results for coupled nonlinear fractional differential equations equipped with nonlocal coupled flux and multi-point boundary conditions*, *Chaos Solitons Fractals*, **102** (2017), 149–161.
- [5] H. H. ALSULAMI, S. K. NTOUYAS, R. P. AGARWAL, B. AHMAD, A. ALSAEDI, *A study of fractional-order coupled systems with a new concept of coupled non-separated boundary conditions*, *Bound. Value Probl.*, (2017) **2017:68**, 1–11.
- [6] J. BASTIEN, *Study of a driven and braked wheel using maximal monotone differential inclusions: applications to the nonlinear dynamics of wheeled vehicles*, *Archive of Applied Mechanics*, **84** (2014), 851–880.
- [7] A. BRESSAN, G. COLOMBO, *Extensions and selections of maps with decomposable values*, *Studia Math.*, **90** (1988), 69–86.
- [8] C. CASTAING, M. VALADIER, *Convex Analysis and Measurable Multifunctions*, *Lecture Notes in Mathematics* **580**, Springer-Verlag, Berlin-Heidelberg-New York, 1977.
- [9] H. COVITZ, S. B. NADLER JR., *Multivalued contraction mappings in generalized metric spaces*, *Israel J. Math.*, **8** (1970), 5–11.
- [10] K. DEIMLING, *Multivalued Differential Equations*, Walter De Gruyter, Berlin-New York, 1992.
- [11] M. F. DANCA, *Synchronization of piecewise continuous systems of fractional order*, *Nonlinear Dynam.*, **78** (2014), 2065–2084.
- [12] Y. DING, Z. WANG, H. YE, *Optimal control of a fractional-order HIV-immune system with memory*, *IEEE Trans. Contr. Sys. Techn.*, **20** (2012), 763–769.
- [13] A. GRANAS, J. DUGUNDJI, *Fixed Point Theory*, Springer-Verlag, New York, 2005.

- [14] M. FAIEGHI, S. KUNTANAPREEDA, H. DELAVARI, D. BALEANU, *LMI-based stabilization of a class of fractional-order chaotic systems*, *Nonlinear Dynam.*, **72** (2013), 301–309.
- [15] J. HENDERSON, N. KOSMATOV, *Eigenvalue comparison for fractional boundary value problems with the Caputo derivative*, *Fract. Calc. Appl. Anal.*, **17** (2014), 872–880.
- [16] S. HU, N. PAPAGEORGIOU, *Handbook of Multivalued Analysis, Volume I: Theory*, Kluwer, Dordrecht, 1997.
- [17] M. JAVIDI, B. AHMAD, *Dynamic analysis of time fractional order phytoplankton-toxic phytoplankton-zooplankton system*, *Ecological Modelling*, **318** (2015), 8–18.
- [18] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, **204**, Elsevier Science B. V., Amsterdam, 2006.
- [19] M. KISIELEWICZ, *Stochastic Differential Inclusions and Applications*, Springer Optimization and Its Applications, **80**, Springer, New York, 2013.
- [20] M. KORDA, D. HENRION, C. N. JONES, *Convex computation of the maximum controlled invariant set for polynomial control systems*, *SIAM J. Control Optim.*, **52** (2014), 2944–2969.
- [21] B. M. B. KRUSHNA, K. R. PRASAD, *On existence of solutions to the Caputo type fractional order three-point boundary value problems*, *Int. J. Anal. Appl.*, **12** (2016), 60–68.
- [22] B. M. B. KRUSHNA, *Eigenvalues for iterative systems of Riemann–Liouville type p -Laplacian fractional-order boundary-value problems in Banach spaces*, *Comp. Appl. Math.*, **39**, **81** (2020), 1–15.
- [23] A. LASOTA, Z. OPIAL, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys.*, **13** (1965), 781–786.
- [24] M. MONTEIRO, D. P. MANUEL, *Differential Inclusions in Nonsmooth Mechanical Problems: Shocks and Dry Friction*, Progress in Nonlinear Differential Equations and their Applications, **9**, Birkhauser Verlag, Basel, 1993.
- [25] I. PETRAS, R. L. MAGIN, *Simulation of drug uptake in a two compartmental fractional model for a biological system*, *Commun. Nonlinear Sci. Numer. Simul.*, **16** (2011), 4588–4595.
- [26] I. PODLUBNY, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [27] K. R. PRASAD, B. M. B. KRUSHNA, *Multiple positive solutions for a coupled system of Riemann–Liouville fractional order two-point boundary value problems*, *Nonlinear Stud.*, **20** (2013), 501–511.
- [28] K. R. PRASAD, B. M. B. KRUSHNA, *Eigenvalues for iterative systems of Sturm–Liouville fractional order two-point boundary value problems*, *Fract. Calc. Appl. Anal.*, **17** (2014), 638–653.
- [29] V. RECUPERO, *A continuity method for sweeping processes*, *J. Differential Equations*, **251** (2011), 2125–2142.
- [30] P. RICHARD, M. NICODEMI, R. DELANNAY, P. RIBIERE, D. BIDEAU, *Slow relaxation and compaction of granular system*, *Nature Mater.*, **4** (2005), 121–128.
- [31] B. SENOL, C. YEROGLU, *Frequency boundary of fractional order systems with nonlinear uncertainties*, *J. Franklin Inst.*, **350** (2013), 1908–1925.
- [32] I. M. SOKOLOV, J. KLAFTER, A. BLUMEN, *Fractional Kinetics*, *Phys. Today*, **55**(2002), 48–54.
- [33] J. R. WANG, Y. ZHANG, *Analysis of fractional order differential coupled systems*, *Math. Methods Appl. Sci.*, **38** (2015), 3322–3338.
- [34] F. ZHANG, G. CHEN, C. LI, J. KURTHS, *Chaos synchronization in fractional differential systems*, *Phil. Trans. R. Soc. A.*, **371** (2013), 20120155.
- [35] Y. ZHOU, B. AHMAD, A. ALSAEDI, *Existence of nonoscillatory solutions for fractional neutral differential equations*, *Appl. Math. Lett.*, **72** (2017), 70–74.