

WEAKLY NONLOCAL BOUNDARY VALUE PROBLEMS WITH APPLICATION TO GEOLOGY

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Abstract. In many cases, groundwater flow in an unconfined aquifer can be simplified to a one-dimensional Sturm-Liouville model of the form:

$$x''(t) + \lambda x(t) = h(t) + \varepsilon f(x(t)), \quad t \in (0, \pi)$$

subject to non-local boundary conditions

$$x(0) = h_1 + \varepsilon \eta_1(x) \text{ and } x(\pi) = h_2 + \varepsilon \eta_2(x).$$

In this paper, we study the existence of solutions to the above Sturm-Liouville problem under the assumption that ε is a small parameter. Our method will be analytical, utilizing the implicit function theorem and its generalizations.

Mathematics subject classification (2010): 34B10, 34B15, 34B09.

Keywords and phrases: Geology, eigenvalue, nonlocal, implicit function theorem, Sturm-Liouville.

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