ON THE EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR FRACTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL MULTI–POINT BOUNDARY CONDITIONS

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Abstract. This paper presents some sufficient conditions for the existence of solutions of fractional differential equation with nonlocal multi-point boundary conditions involving Caputo fractional derivative and integral boundary conditions. Our analysis relies on the Banach contraction principle, Boyd and Wong fixed point theorem, Leray-Schauder nonlinear alternative. Finally, examples are provided to illustrate our main results.

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