

## EXISTENCE AND CONTINUATION OF SOLUTIONS OF HILFER–KATUGAMPOLA–TYPE FRACTIONAL DIFFERENTIAL EQUATIONS

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*Abstract.* In this article, we discuss the existence and continuation of the solution to generalized Cauchy-type problem involving Hilfer-Katugampola-type fractional derivative. Firstly, we establish new theorems of local existence by using Schauder fixed point theorem. Then, we deduce continuation theorems for general fractional differential equations. By applying continuation theorems, we present several global existence results. Moreover, the examples are given to illustrate our main results.

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