Differential
Equations
& Applications
Volume 13, Number 4 (2021), 355–371
doi:10.7153/dea-2021-13-20

STABILITY OF NONAUTONOMOUS IMPULSIVE EVOLUTION SYSTEM ON TIME SCALE

AKBAR ZADA, YASIR ARAFAT AND SYED OMAR SHAH*

Abstract. The main theme of this article is to discuss the existence, uniqueness and β–Ulam type stability for nonautonamous impulsive differential systems on time scale by applying fixed point method. The major components to proof the results are the Grönwall inequality on time scale, abstract Grönwall lemma and Picard operator. Some suppositions are made for achieving our results. At last, the main result is validated by the example specified in this paper.

Mathematics subject classification (2020): 26D15, 26A51, 32F99, 41A17.
Keywords and phrases: Stability, time scale, fixed point, impulses, dynamic system.

REFERENCES

[1] R. P. AGARWAL, A. S. AWAN, D. ÓREGAN AND A. YOUNUS, *Linear impulsive Volterra integro–dynamic system on time scales*, Adv. Difference Equ., **2014**, 2014: 6.

[2] C. ALSINA AND R. GER, *On some inequalities and stability results related to the exponential function*, J. Inequal. Appl., **2** (1998), 373–380.

[3] S. ANDRÁS AND A. R. MÉSZÁROS, *Ulam–Hyers stability of dynamic equations on time scales via Picard operators*, Appl. Math. Comput., **209** (2013), 4853–4864.

[4] D. D. BAINOV AND A. DISHLIEV, *Population dynamics control in regard to minimizing the time necessary for the regeneration of a biomass taken away from the population*, Comp. Rend. Bulg. Scie., **42** (1989), 29–32.

[5] D. D. BAINOV AND P. S. SIMENOV, *Systems with impulse effect stability theory and applications*, Ellis Horwood Limited, Chichester, UK, (1989).

[6] M. BOHNER AND A. PETERSON, *Dynamic equations on time scales: an introduction with applications*, Birkhäuser, Boston, Mass, USA, (2001).

[7] M. BOHNER AND A. PETERSON, *Advances in dynamics equations on time scales*, Birkhäuser, Boston, Mass, USA, (2003).

[8] J. J. DACHUNHA, *Stability for time varying linear dynamic systems on time scales*, J. Comput. Appl. Math., **176** (2) (2005), 381–410.

[9] A. HAMZA AND K. M. ORABY, *Stability of abstract dynamic equations on time scales*, Adv. Difference Equ., **2012**, 2012: 143.

[10] S. HILGER, *Analysis on measure chains–A unified approach to continuous and discrete calculus*, Result math., **18** (1–2) (1990), 18–56.

[11] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A., **27** (4) (1941), 222–224.

[12] S.-M. JUNG, *Hyers–Ulam stability of linear differential equations of first order*, Appl. Math. Lett., **17** (10) (2004), 1135–1140.

[13] S.-M. JUNG, *Hyers–Ulam–Rassias stability of functional equations in nonlinear analysis*, Springer Optim. Appl., Springer, NewYork, **48**, (2011).

[14] Y. LI AND Y. SHEN, *Hyers–Ulam stability of linear differential equations of second order*, Appl. Math. Lett., **23** (3) (2010), 306–309.

[15] T. LI AND A. ZADA, *Connections between Hyers–Ulam stability and uniform exponential stability of discrete evolution families of bounded linear operators over Banach spaces*, Adv. Difference Equ., **2016**, 2016: 153.

© ELEMENT, Zagreb
Paper DEA-13-20

[16] V. LUPULESCU AND A. ZADA, *Linear impulsive dynamic systems on time scales*, Electron. J. Qual. Theory Differ. Equ., **2010** (11) (2010), 1–30.

[17] S. I. NENOV, *Impulsive controllability and optimization problems in population dynamics*, Nonlinear Anal. Theory Methods Appl., **36** (7) (1999), 881–890.

[18] M. OBŁOZA, *Hyers stability of the linear differential equation*, Rocznik Nauk.-Dydakt. Prace Mat., (1993), 259–270.

[19] M. OBŁOZA, *Connections between Hyers and Lyapunov stability of the ordinary differential equations*, Rocznik Nauk.-Dydakt. Prace Mat., **14** (1997), 141–146.

[20] C. PÖTZSCHE, S. SIEGMUND AND F. WIRTH, *A spectral characterization of exponential stability for linear time–invariant systems on time scales*, Discrete Contin. Dyn. Sys., **9** (5) (2003), 1223–1241.

[21] T. M. RASSIAS, *On the stability of linear mappings in Banach spaces*, Proc. Amer. Math. Soc., **72** (2) (1978), 297–300.

[22] R. RIZWAN, J. R. LEE, C. PARK AND A. ZADA, *Switched coupled system of nonlinear impulsive langevin equations with mixed derivatives*, AIMS Math., **6** (12) (2021), 13092–13118.

[23] R. RIZWAN AND A. ZADA, *Nonlinear impulsive Langevin equation with mixed derivatives*, Math. Methods Appl. Sci., **43** (1) (2020), 427–442.

[24] R. RIZWAN AND A. ZADA, *Existence theory and Ulam's stabilities of fractional Langevin equation*, Qual. Theory Dyn. Syst., **20** (2) (2021), 1–17.

[25] R. RIZWAN, A. ZADA, M. AHMAD, S. O. SHAH AND H. WAHEED, *Existence theory and stability analysis of switched coupled system of nonlinear implicit impulsive Langevin equations with mixed derivatives*, Math. Methods Appl. Sci., **44** (11) (2021), 8963–8985.

[26] R. RIZWAN, A. ZADA AND X. WANG, *Stability analysis of non linear implicit fractional Langevin equation with non–instantaneous impulses*, Adv. Differ. Equ., **2019** (85) (2019), 1–31.

[27] I. A. RUS, *Grönwall lemmas: ten open problems*, Sci. Math. Jpn., **70** (2009), 221–228.

[28] S. ŞEVGIN AND H. ŞEVLI, *Stability of a nonlinear Volterra integro-differential equation via a fixed point approach*, J. Nonlinear Sci. Appl., **9** (1) (2016), 200–207.

[29] R. SHAH AND A. ZADA, *A fixed point approach to the stability of a nonlinear volterra integrodifferential equations with delay*, Hacettepe J. Math. Stat., **47** (3) (2018), 615–623.

[30] S. O. SHAH AND A. ZADA, *On the stability analysis of non–linear Hammerstein impulsive integro–dynamic system on time scales with delay*, Punjab Univ. J. Math., **51** (7) (2019), 89–98.

[31] S. O. SHAH AND A. ZADA, *Existence, uniqueness and stability of solution to mixed integral dynamic systems with instantaneous and noninstantaneous impulses on time scales*, Appl. Math. Comput., **359** (2019), 202–213.

[32] S. O. SHAH, A. ZADA AND A. E. HAMZA, *Stability analysis of the first order non-linear impulsive time varying delay dynamic system on time scales*, Qual. Theory Dyn. Syst., **18** (3) (2019), 825–840.

[33] S. O. SHAH, A. ZADA, M. MUZAMIL, M. TAYYAB AND R. RIZWAN, *On the Bielecki–Ulam's type stability results of first order non-linear impulsive delay dynamic systems on time scales*, Qual. Theory Dyn. Syst., **19** (98) (2020), 1–18.

[34] S. O. SHAH, A. ZADA, C. TUNC AND A. ALI, *Bielecki–Ulam–Hyers stability of non-linear Volterra impulsive integro-delay dynamic systems on time scales*, Punjab Univ. J. Math., **53** (5) (2021), 339–349.

[35] S. M. ULAM, *A collection of the mathematical problems*, Interscience Publisheres, New York-London, (1960).

[36] S. M. ULAM, *Problem in modern mathematics*, Science Editions, J. Wiley and Sons, Inc., Nework, (1964).

[37] J. WANG, M. FEČKAN AND Y. TIAN, *Stability analysis for a general class of non-instantaneous impulsive differential equations*, Mediterr. J. Math., **14** (46) (2017), 1–21.

[38] J. WANG, M. FEČKAN AND Y. ZHOU, *Ulam's type stability of impulsive ordinary differential equations*, J. Math. Anal. Appl., **395** (2012), 258–264.

[39] J. WANG, M. FEČKAN AND Y. ZHOU, *On the stability of first order impulsive evolution equations*, Opuscula Math., **34** (3) (2014), 639–657.

[40] J. WANG AND X. LI, *A uniform method to Ulam–Hyers stability for some linear fractional equations*, Mediterr. J. Math., **13** (2016), 625–635.

[41] J. WANG, A. ZADA AND W. ALI, *Ulam's–type stability of first–order impulsive differential equations with variable delay in quasi–Banach spaces*, Int. J. Nonlin. Sci. Num., **19** (5) (2018), 553–560.

[42] J. Wang and Y. Zhang, *A class of nonlinear differential equations with fractional integrable impulses*, Com. Nonl. Sci. Num. Sim., **19** (2014), 3001–3010.

[43] A. Younus, D. O'Regan, N. Yasmin and S. Mirza, *Stability criteria for nonlinear Volterra integro–dynamic systems*, Appl. Math. Inf. Sci., **11** (5) (2017), 1509–1517.

[44] X. Yu, J. Wang and Y. Zhang, *On the β–Ulam–Hyers–Rassias stability of nonautonomous impulsive evolution equations*, J. Appl. Math. Comput., **48** (2015), 465–475.

[45] A. Zada and S. Ali, *Stability Analysis of Multi-point Boundary Value Problem for Sequential Fractional Differential Equations with Non-instantaneous Impulses*, Int. J. Nonlinear Sci. Numer. Simul., **19** (7) (2018), 763–774.

[46] A. Zada, W. Ali and S. Farina, *Hyers–Ulam stability of nonlinear differential equations with fractional integrable impulses*, Math. Methods Appl. Sci., **40** (15) (2017), 5502–5514.

[47] A. Zada, R. Rizwan, J. Xu and Z. Fu, *On implicit impulsive Langevin equation involving mixed order derivatives*, Adv. Differ. Equ., **2019** (489) (2019), 1–26.

[48] A. Zada, S. O. Shah and Y. Li, *Hyers–Ulam stability of nonlinear impulsive Volterra integro–delay dynamic system on time scales*, J. Nonlinear Sci. Appl., **10** (11) (2017), 5701–5711.

[49] A. Zada, O. Shah and R. Shah, *Hyers–Ulam stability of non-autonomous systems in terms of boundedness of Cauchy problems*, Appl. Math. Comput., **271** (2015), 512–518.

[50] A. Zada and S. O. Shah, *Hyers–Ulam stability of first–order non–linear delay differential equations with fractional integrable impulses*, Hacettepe J. Math. Stat., **47** (5) (2018), 1196–1205.

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com