THREE DIMENSIONAL SYSTEM OF GLOBALLY MODIFIED MAGNETOHYDRODYNAMICS EQUATIONS WITH INFINITE DELAYS

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Abstract. Existence and uniqueness of strong solutions for three dimensional system of globally modified magnetohydrodynamics equations containing infinite delays terms are established together with some qualitative properties of the solution in this work. The existence is proved by making use of Galerkin's method, Cauchy-Lipshitz's theorem, a priori estimates, the Aubin-Lions compactness theorem. Moreover, we study the asymptotic behavior of the solution.

Mathematics subject classification (2020): 26D15, 26A51, 32F99, 41A17.
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