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A BOUND FOR ZEROS OF SOLUTIONS TO A HIGHER ORDER NON–HOMOGENEOUS ODE WITH POLYNOMIAL COEFFICIENTS

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Abstract. Let $P_k(z)\ (k=1,2,\dots,n)$ and $G(z)$ be polynomials with complex in general coefficients. The paper deals with the higher order differential equation

$$v^{(n)}(z)+P_1(z)v^{(n-1)}(z)+\ldots+P_n(z)v(z)=G(z).$$

We derive estimates for the sums of the zeros of solutions to this equation. These estimates give us bounds for the function counting the zeros of solutions and information about the zero-free domain. Some other applications are also discussed.

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