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SHARP WELL–POSEDNESS AND ILL–POSEDNESS RESULTS FOR DISSIPATIVE KDV EQUATIONS ON THE REAL LINE

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Abstract. This work is concerned about the Cauchy problem for the following generalized KdV-Burgers equation

$$\begin{cases} \partial_t u + \partial_x^3 u + L_p u + u\partial_x u = 0, \\ u(0,x) = u_0(x), \end{cases}$$

where L_p is a dissipative multiplier operator. Using Besov-Bourgain Spaces, we establish a bilinear estimate and following the framework developed in [14] we prove sharp local and global well-posedness in the Sobolev spaces $H^{-p/2}(\mathbb{R})$ and ill-posedness in $H^s(\mathbb{R})$ when $s < -p/2$, both when $p \geqslant 2$. Also, we prove C^2 -ill-posedness in $H^s(\mathbb{R})$, for $s < 3/2 - p/4$ and $0 \leqslant p \leqslant 2$.

Mathematics subject classification (2020): 35E15, 35M11, 35Q53, 35Q60.
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